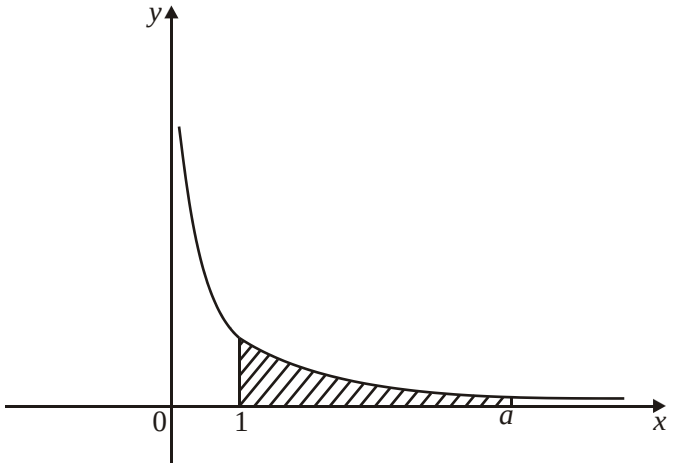
IB Integration practice questions

1. The diagram shows part of the graph of $y = \frac{1}{x}$. The area of the shaded region is 2 units.



Find the exact value of a.

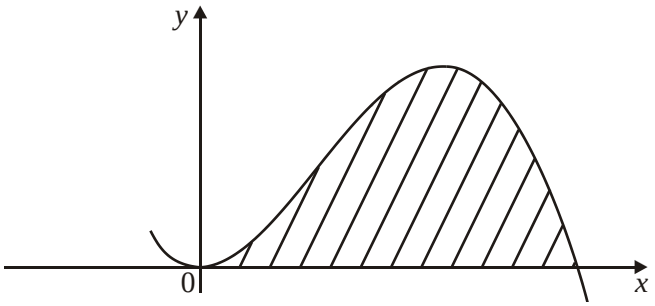
Working:

Answer:

.....

(Total 4 marks)

2. The diagram shows part of the graph of $y = 12x^2(1 - x)$.



(a) Write down an integral which represents the area of the shaded region.

(b) Find the area of the shaded region.

Working:

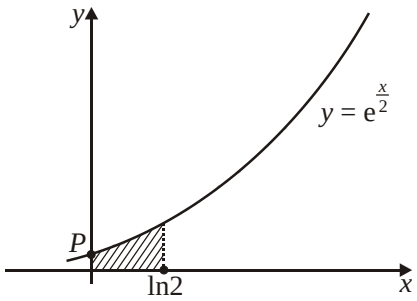
Answers:

(a) .....

(b) .....

(Total 4 marks)

3. The diagram shows part of the graph of $y = e^{\frac{x}{2}}$.



(a) Find the coordinates of the point P, where the graph meets the y-axis.

(2)

The shaded region between the graph and the x-axis, bounded by $x = 0$ and $x = \ln 2$, is rotated through 360° about the x-axis.

(b) Write down an integral which represents the volume of the solid obtained.

(4)

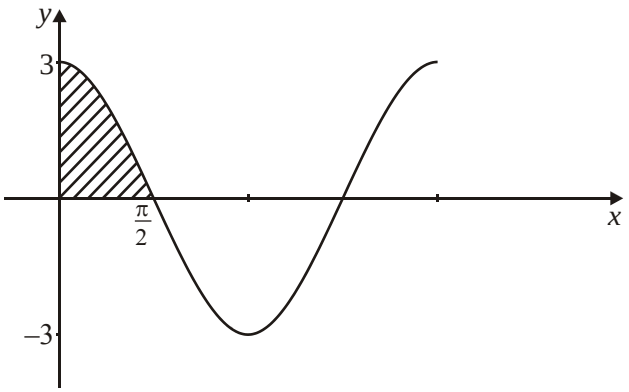
(c) Show that this volume is π.

(5)

(Total 11 marks)

4. The graph represents the function

$$f : x \mapsto p \cos x, p \in \mathbb{N}.$$



Find

(a) the value of p;

(b) the area of the shaded region.

Working:

Answers:

(a) .....

(b) .....

(Total 4 marks)

5. If $f'(x) = \cos x$, and $f\left(\frac{\pi}{2}\right) = -2$, find $f(x)$.

Working:

Answer:

.....

(Total 4 marks)

6. Find

(a) $\int \sin(3x+7)\mathrm{d}x$;

(b) $\int e^{-4x}\mathrm{d}x$.

Working:

Answers:

(a) .....

(b) .....

(Total 4 marks)

7. In this question, s represents displacement in metres, and t represents time in seconds.

(a) The velocity v m s^{-1} of a moving body may be written as $v = \frac{ds}{dt} = 30 - at$, where a is a constant. Given that $s = 0$ when $t = 0$, find an expression for s in terms of a and t.

(5)

Trains approaching a station start to slow down when they pass a signal which is 200 m from the station.

(b) The velocity of Train 1 t seconds after passing the signal is given by $v = 30 - 5t$.

(i) Write down its velocity as it passes the signal.

(ii) Show that it will stop before reaching the station.

(5)

(c) Train 2 slows down so that it stops at the station. Its velocity is given by $v = \frac{ds}{dt} = 30 - at$, where a is a constant.

(i) Find, in terms of a, the time taken to stop.

(ii) Use your solutions to parts (a) and (c)(i) to find the value of a.

(5)
(Total 15 marks)